Integer Multiplication

Question:

How do you know that the answer to (-3) x (-2) is +6?

Possible answers to the question are:

1. I was told two minuses make a plus and I know 3 x 2 is 6. So, the answer must be plus six.
2. If you have negative 3 groups of negative 2, the negatives cancel, leaving 6 as the answer.
3. Why would anyone want to multiply minus numbers? What's the point?

Response 1 is a procedural response that does not reveal any understanding about why the answer is 2. As well, two minuses do not always make a plus.

For example, $-a^{-2}$ is not the same as a^2 The two minuses do not make a plus.

Response 2 is a procedural response. It does attempt to provide an explanation using the meaning of multiplication as 'groups of' and the notion of canceling (likely borrowed from integer addition/subtraction). The explanation is faulty. One reason it is faulty is because you cannot have a negative group. A group, such as a bag, person, or van, always involves a count of how many groups there are, and a count must be a positive whole number or zero. For example, for 3 bags with 6 candies in each bag, the '3' represents a count of how many bags (the number of groups, in other words) and that count must be positive.

Response 3 asks a pointed question. Where does integer multiplication occur? Certainly not in the everyday world of most people's experiences. Integer multiplication typically happens in the domain of science, engineering, and technology, and that world is hidden from the eyes of most people.

Models for teaching Integer Multiplication

Three models are useful:

- Gaining and losing
- Two-colour counters
- Number line
- Patterning

Gaining and losing, two-colour counters, and the number line are useful for the cases: positive x positive and positive x negative. A mathematical principle is useful for the case: negative x positive and patterning is useful for the cases: negative x positive and negative x negative. Each case is considered in turn.

Case: Positive x Positive

This case is easy because it is basically a revisit of whole numbers multiplication.

Gaining and losing

A context such as Paul increased in height by 2 cm for each of 3 months should make clear to students that positive x positive = positive.

Two-colour counters

Agree that, say red = positive and blue = negative. Then lay out three piles of red counters with five in each pile. This should make clear to students that **positive x positive = positive**.

Number line

This involves repeated addition on the number line. For example, to obtain the answer to 2 x 3, start at zero and take 2 steps of 3 to the right. The answer is where you end up (positive 6). For detail on integer addition, refer to: Grade 7 Integer addition

Case: Positive x negative

This case is also easy because it involves a group count that is positive. [The first number of the multiplication represents the group count. Refer to The meanings of the arithmetic operations for details.]

Gaining and losing

A context such as Paul lost $2 each month for 3 months (3 x (-2)) should make clear to students that positive x negative = negative.

Two-colour counters

Agree that, say red = positive and blue = negative. Then lay out three piles of blue counters with five in each pile (3 x (-5)). This should make clear to students that **positive x negative = negative**.

Number line

This involves repeated addition on the number line. For example, to obtain the answer to 2 x (-3), start at zero and take 2 steps of 3 to the left. The answer is where you end up (negative 6). For detail on integer addition, refer to: Grade 7 Integer addition

Case: Negative x Positive

Gaining and losing, two-colour counters, and the number line cannot be used for this case because the first number (the group count) is negative. A group count must be positive.

Mathematical principle (commutative for x: switching is okay when multiplying)

The assumption is that students understand the case: positive x negative. Suppose the question is 2 x (-3). Students know the answer to this is -6. In whole number multiplication, 2 x 3 = 3 x 2. In other words switching the numbers gives the same result when multiplying. This also is true for integers. Thus, for example, 2 x (-3) and (-3) x 2 result in the same product. Thus negative x positive = negative.

Patterning

The assumption is that students understand the case: positive x positive = positive.

Consider the multiplication series shown here.

3 x 2 = 6
2 x 2 = 4
1 x 2 = 2
0 x 2 = 0
(-1) x 2 = ?

There is a pattern, namely, the product gets smaller by 2 each time. The next product should be -2.

In other words, negative x positive = negative.

Case: Negative x Negative

Gaining and losing, two-colour counters, and the number line cannot be used for this case because the first number (the group count) is negative. A group count must be positive. Only the patterning model is useful for this case.

Patterning

The assumption is that students understand the case: positive x negative = negative.

Consider the multiplication series shown here.

There is a pattern, namely, the product gets bigger by 2 each time. The next product should be +2.

In other words, negative x negative = positive.

$3 \times (-2) = -6$
$2 \times (-2) = -4$
$1 \times (-2) = -2$
$0 \times (-2) = 0$
$(-1) \times (-2) = ?$